Calculation of Value-at-Risk Variance-Covariance with the Approach of Simple Cash Portfolio, Factor Models and Cash Flow

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Abstract

One way to calculate Value-at-Risk (VaR) is the variation-covariance method. The calculation of VaR covariance assumes stock data is normally distributed. The data needed to calculate VaR by the variance-covariance method is the covariance matrix of Bank Danamon and Bank Mandiri stock data. The main topics discussed in this paper are calculating VaR covariance with a simple cash portfolio approach, factor models and cash flow. For comparison of the use of the three approaches Backtesting, the backtest results indicate that the factor model is the best method.

Keywords: Value at Risk, variance-covariance, simple cash portfolio, factor models, cash flow.

1. Introduction

Financial activity is very volatile and risky. Risk can be interpreted as the possibility of undesired or opposite results being desired. To avoid losses, investors should be able to face all the risks that might occur to obtain the results as expected and can also calculate these risks (Fabozzi, 2000; Jogiyanto. 2007).

To control and calculate these risks, you can use Value at Risk (commonly abbreviated as VaR) which has many advantages, one of the main advantages of VaR is that it does not neglect the underlying risk factors and specific risks of the portfolio (Sukono et al., 2019). VaR can be used to compare market risks from all types of activities in a company and provide a single calculation that is easily understood. VaR can also be widely used and has become a standard in risk calculation because it can be applied to all types of risks. By using VaR, investors can calculate investment risk. One method of calculating VaR is variance and covariance. The covariance method has one advantage, i.e. it is very fast and easy to calculate VaR (Redhead, 1997).

2. Methodology

2.1. Normal Distribution and Normal VaR Estimation

Normal distribution is often also called a Gaussian distribution. This normal distribution depends on three variables, namely the x value, the average distribution notated by μ, and the standard deviation notated by σ.

Definition 1 (Mood et al., 1963)**.** The random variable X which has a mean μ and variance $\sigma^2 < \infty$ is said to be normally distributed, abbreviated $X \sim N(\mu, \sigma^2)$ if the probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty. \tag{1}$$

Calculation of probability in the normal distribution model will involve searching for the integral value

$$\int_{-\infty}^{a} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

for each real number a. This integral can be simplified through the following theorem (Sudjana, 2005).

Theorem 1. If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$.

Proof:

The Z distribution function is

$$G(z) = P(Z \le z) = P\left(\frac{x-\mu}{\sigma} \le z\right) = P(X \le \mu + \sigma z) = \int_{-\infty}^{\mu+\sigma z} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

Writing $t = \frac{X-\mu}{\sigma}$, so

$$G(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt, \text{ this shows that } Z \sim N(0, 1).$$

So that for each real number a and b apply

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right) = P\left(Z \le \frac{b-\mu}{\sigma}\right) - P\left(Z \le \frac{a-\mu}{\sigma}\right)$$

$Z \sim N(0, 1)$ means a normal distribution that has a mean of 0 and a variance of 1. If Z is a normal standard, then the distribution of $\mu + \sigma Z$ is called a normal distribution with mean μ and variance σ^2 and is denoted by $N(\mu, \sigma^2)$ (Walpole and Myers, 1978). ∎

Based on the above normal distribution, so that the estimation for normal VaR can be written as follows

$$\Delta_k P_t \sim N(\mu_t, \sigma_t^2)$$

$$\frac{\Delta_k P_t - \mu}{\sigma} \sim N(0,1),$$

where P_t is the stock price at time t, and

$$\Delta_h P_t = P_{t+h} - P_t.$$

VaR of $100\alpha\%$ D-day is

$$\text{Prob}(\Delta_h P_t < -VaR_{\alpha,h}) = \alpha.$$

Normal standard transformation is

$$\text{Prob}\left(\frac{\Delta_h P_t - \mu_t}{\sigma_t} < \frac{-VaR_{\alpha,h} - \mu_t}{\sigma_t}\right) = \alpha$$

and

$$\text{Prob}\left(Z_t < \frac{-VaR_{\alpha,h} - \mu_t}{\sigma_t}\right) = \alpha.$$

Based on the VaR definition,

$$\text{Prob}(Z_t < Z_\alpha) = \alpha,$$

then

$$\frac{-VaR_{\alpha,h} - \mu_t}{\sigma_t} = Z_\alpha$$

$$VaR_{\alpha,h} = -Z_\alpha \times \sigma_t - \mu_t,$$

where $\mu_t = 0$, because VaR is only suitable for calculating short-term risks. Thus, the VaR value can be calculated by the formula (Dowd, 2002):

$$VaR = -Z_\alpha \times \sigma_t. \qquad (2)$$

2.2. Estimation of VaR Variance-Covariance

The approach used to estimate VaR covariance is a simple cash portfolio, factor models and cash flow.

2.2.1. Simple Cash Portfolio

If $\mathbf{p'Vp}$ represents variance, then the VaR volatility value can be calculated with

$$\sigma = (\mathbf{p'Vp})^{\frac{1}{2}}. \qquad (3)$$

So the equation for the VaR estimate becomes (Dowd, 2002):

$$VaR = -Z_\alpha (\mathbf{p'Vp})^{\frac{1}{2}}. \qquad (4)$$

2.2.2. VaR Estimation with Factor Models

Factor models are statistical approaches that can be used to analyze the interrelationship of several variables and to explain what dimensions (called factors) underlie these variables and reduce them (Sukono et al., 2019).

The factor model is usually used for large scale equity portfolios. Suppose a large-scale equity portfolio with k assets, which is denoted by a factor model with m risk factors. The general equation for the factor model is (Klienbaum et al., 1988):

$$R_{it} = \alpha_i + \beta_{i1} f_{1t} + \Lambda + \beta_{im} f_{mt} + \varepsilon_i, \qquad t = 1, 2, \text{K}, T, \quad i = 1, 2, \text{K}, k. \qquad (5)$$

From the above equation, the VaR calculation can be done using the following equation:

$$\text{variance due to market risk factors} = \mathbf{p'BV}_x\mathbf{B'p}, \qquad (6)$$

$$\text{variance specific} = \mathbf{p'V}_\varepsilon\mathbf{p}, \qquad (7)$$

$$\text{variance total} = \mathbf{p'BV}_x\mathbf{B'p} + \mathbf{p'V}_\varepsilon\mathbf{p}, \qquad (8)$$

$$VaR = -Z_\alpha (\mathbf{p'BV}_x\mathbf{B'p} + \mathbf{p'V}_\varepsilon\mathbf{p})^{\frac{1}{2}}. \qquad (9)$$

2.2.3. VaR estimation with cash flow map

In addition to simple cash portfolio and factor models, another approach that can be used to calculate VaR covariance is cash flow. Cash flow is the amount of balance of cash received and paid by the company during a certain period.

Cash flow maps can be used to calculate VaR covariance from loan portfolios, i.e.

$$\mathbf{p} = \left(PV_1, PV_2, \text{K}, PV_k\right), \tag{10}$$

$$PV = \frac{C}{\left(1 + r_1\right)\left(1 + r_2\right)}. \tag{11}$$

The VaR of the original cash flow will not be the same as the VaR of the mapped cash flow. The variance of the original cash flow is the same as the variance of the portfolio, which can be written into the equation (Ruppert, 2004):

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2\left(w_1 w_2 p_{1,2} \sigma_1 \sigma_2\right) \tag{12}$$

2.3. Backtest

This section will discuss the backtest that will be used to evaluate the performance of the VaR model. Backtest is an important part of the risk measurement process. In the process, the backtest has four important elements and a single output. The first element is a collection of n observations of loss or profit for each period that are interconnected with VaR forecasts. The second element is the value of the loss function depends on VaR forecasting in its period.

The backtest used in this paper is Dowd (2002) which states the loss function as follows:

$$C_t = \begin{cases} \left(L_t - VaR_t\right)/VaR_t, & L_t > VaR_t \\ 0, & L_t \leq VaR_t \end{cases} \tag{13}$$

where L_t is loss (profit if negative) in the form of return (r_t).

The third element is the benchmark value, which is the expected value of the best model. The fourth element is the value of the function, which is taken as the input of the loss function and the reference value. If the C_t reference value uses the null hypothesis, the QPS (quadratic probability score) value is

$$QPS = \frac{2}{n} \sum_{t=1}^{n} \left(C_t - p\right)^2, \tag{14}$$

where $p = 1 - \alpha$ and n are lots of stock data. The range of QPS values is at [0, 2]. For QPS values close to zero, the model is getting better.

3. Results and Discussion

For example, the initial investment value at Bank Danamon and Bank Mandiri is 1,00 IDR. Using a 95% confidence coefficient, Bank Danamon's share variance of 0.000250 and Bank Mandiri of 0.000241, and covariance of both shares of 0.000245, VaR will be calculated for each share with the three approaches (Sulaiman, 2003).

3.1. Calculation of VaR Covariance with Simple Cash Portfolio for Single Shares and Portfolios

Using equation (3), the volatility value for each stock is obtained

$$\sigma_p = \left(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.000250 & 0.000245 \\ 0.000245 & 0.000241 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^{\frac{1}{2}} = 0.0315.$$

Then calculate VaR using equation (4), so the following results are obtained

$$VaR_p = 1.645 \times 0.0315 = 0.0518 \text{IDR}.$$

3.2. Calculation of VaR Covariance with a Factor Model for Single Shares and Portfolios

The market shares used are the CSPI (IHSG) and the exchange rate of the Rupiah (IDR) to the Euro. IHSG variance is 0.000113 and 0.0000187 for the exchange rate, both covariance are 0.0000459. **B** is the matrix $m \times k$, each element is a beta of the i-th stock and the j-th risk factor.

The value of variance for company shares and market shares is

$$\sigma^2_{I,L} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 3.332 & -0.805 \\ 1.174 & -0.173 \end{bmatrix} \begin{bmatrix} 0.000113 & 0.000459 \\ 0.000459 & 0.000187 \end{bmatrix} \begin{bmatrix} 3.332 & -0.805 \\ 1.174 & -0.173 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0.0019069$$

Then, obtained VaR for each stock using equation (9)

$$VaR_p = 1.645 \times (0.0019069 + 0.000992)^{\frac{1}{2}} = 0.088569 \text{IDR}.$$

3.3. Calculation of VaR Covariance with Cash Flow for Single Shares and Portfolios

For example, cash flow on each Bank Danamon and Bank Mandiri shares is 1,00 IDR. r is the annual interest rate on SBI. Calculate the present value for each share using Equation (11), so that the VaR value is obtained as follows

$$VaR_p = 1.645 \times 0.0143763 = 0.0236490 \text{IDR}.$$

3.4. Backtesting

After the VaR of each share is obtained, calculate the value of Ct using equation (13). Then, the QPS for each approach is as follows.

1) Backtest calculation for simple cash portfolio:

$$QPS_p = \frac{2}{476} \times 1.383069 = 0.005811$$

2) Backtest calculation for factor models:

$$QPS_p = \frac{2}{476} \times 1.19 = 0.005.$$

3) Backtest calculation for cash flow:

$$QPS_p = \frac{2}{476} \times 13.07707 = 0.0549460$$

4. Conclussion

VaR calculation with cash flow results in a smaller risk value than the other two methods. Whereas the calculation of VaR with the factor model produces the greatest VaR. This indicates that market shares have an influence on the VaR calculation. With market influence, the risk of loss also increases.

VaR calculation using the factor model approach is better than a simple cash portfolio and cash flow. This can be seen from the backtest calculation for each method, from the backtest calculation results, the QPS value of the factor model is smaller than the simple cash portfolio and cash flow.

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