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Bankruptcy Probability Analysis of PT XYZ Using a Heavy-Tail (Pareto) Discrete Surplus Model

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Abstract

The insurance industry plays a strategic role in maintaining societal financial stability by providing protection mechanisms against unforeseen risks. However, the risk of bankruptcy remains a real threat when policyholder claims exceed the company's reserve funds and collected premiums. This necessitates a quantitative approach capable of projecting bankruptcy probability more accurately. This study is designed to analyze the bankruptcy probability of PT XYZ by utilizing a discrete surplus model based on the heavy-tail Pareto distribution. This model was selected due to its characteristics, which can effectively represent large, infrequent claims that nonetheless have a significant impact on the company's financial condition. The research data will be sourced from the company's financial reports and used in the bankruptcy probability modeling process employing the Pareto distribution approach. This research is expected to provide a theoretical contribution by enriching actuarial literature, particularly concerning the application of heavy-tail surplus models in bankruptcy risk analysis. It also aims to offer practical benefits for insurance companies in designing more comprehensive risk management strategies. Furthermore, the study's findings are hoped to provide valuable input for regulators in strengthening policyholder protection policies and supporting the stability of the national insurance industry.

Keywords: Bankruptcy, Insurance, Surplus model, Heavy-tail, Pareto Distribution

1. Introduction

The insurance industry has a strategic role in maintaining people's financial stability through the provision of protection mechanisms against various unforeseen risks. Risk of bankruptcy (*Ruin*) arises when the number of claims filed by the policyholder exceeds the reserve funds held by the company (Erizal & Septiadi, 2021). This condition indicates the importance of a quantitative approach that is able to project the level of risk more accurately and reliably.

The phenomenon of insurance company bankruptcy not only has an impact on the company's operational sustainability, but also has significant consequences for policyholders as parties who depend on protection guarantees. The company's inability to meet its claims payment obligations has implications for a decline in public trust in the insurance industry. The surplus model is seen as a relevant analytical instrument for measuring the probability of bankruptcy because it is able to represent the dynamics of the interaction between premiums, claims, and reserves in a more systematic manner.

Implementation of the surplus model with distribution *heavy-tail*, especially Pareto, is considered relevant in modeling large claims that often occur in the insurance sector (Qin & Jacob, 2022). This distribution reflects the phenomenon of extreme losses that are difficult to accommodate with conventional distributions such as normal and exponential. Studies on the application of this model in developing countries, including Indonesia, are still very limited. This is significant considering the characteristics of the Indonesian insurance market which is characterized by a low penetration rate and regulatory complexity, so it requires further research attention.

Previous studies have focused more on the solvency approach with the *Value at Risk* (VaR) and numerical simulations. For example, Erizal & Septiadi (2021) conducting a bankruptcy risk analysis through simulations, but not yet directly attributing it to the distribution *heavy-tail* Pareto. The model used is still limited to exponential and Poisson distributions so it has not taken into account the characteristics of extreme claims. This confirms that there is a research gap that needs to be bridged through a more relevant approach.

This study was designed to analyze the probability of bankruptcy of PT XYZ by utilizing a discrete surplus model based on the Pareto *heavy-tail* distribution . The research contribution is expected to enrich the actuarial literature by presenting a more representative approach in describing large claims and providing practical recommendations for insurance companies in designing risk management strategies. In addition, this research is expected to have positive implications for regulators in strengthening policies oriented towards protecting the interests of the public and the stability of the national insurance industry.

2. Literature Review

2.1 Insurance Concept and Bankruptcy Risk

The basic concept of insurance places this industry as a financial mechanism that functions to transfer risk from the insured to the insurer. Mechanism *pooling of risk* allows insurers to collect premiums from multiple participants to pay for the claims of a small portion of those who suffer losses. This function not only provides financial protection for individuals or entities, but also maintains macroeconomic stability by reducing uncertainty in business activities and people's lives (Eka Tripustikasari, 2025).

The risk of bankruptcy in insurance arises when the claims obligations that a company must pay exceed the capital capacity, reserve funds, and available assets. This condition causes the company to be in a position of insolvency that threatens operational sustainability. The level of solvency has been shown to be directly related to the stability of the financial system, so a low solvency ratio increases the vulnerability of companies to financial failure (Kopong & Balun, 2023). Supervision of capital structure is a crucial aspect in efforts to maintain the resilience of the insurance industry.

Financial ratios as an early prediction tool have proven to be effective in detecting the possibility of bankruptcy in insurance companies. Frequently used indicators among *lain Debt to Equity Ratio* (DER), *Risk-Based Capital* (RBC), liquidity ratio, and profitability level. A high DER reflects the risk of inability to meet obligations, while a low RBC indicates a weak capital to cover claims (Putra & Trisnaningsih, 2021). Financial ratio-based quantitative measurement provides an objective basis for risk management decision-making and is an important instrument in maintaining the sustainability of the insurance business.

2.2 Discrete Time Surplus Model

The discrete time surplus model describes the insurance company's capital reserves in each period as a combination of initial capital, premium receipts, and random claims. Surplus in the second period is defined as:t

$$U_t = u + \sum_{j=1}^{t} (P_j + C_j - S_j) = U_{t-1} + P_t + C_t - S_t, \tag{1}$$

with,

U_t : surplus at the end of the second period,t
u : initial capital,
P_t : premiums collected in the fourth period,t
C_t : other cash flows,
S_t : the number of claims paid in the second period t(Klugman *et al*., 2019).

This model assumes that random variables are only affected by the surplus of the previous period $W_t = P_t + C_t - S_t U_{t-1}$, so that $\{U_t\}$ they form a Markov process.

The advantage of discrete models over continuous models lies in flexibility in empirical application, particularly when premium and claims data are recorded at discrete time intervals such as monthly or yearly. This model is also relevant to use in insurance industry practices because it is able to represent the financial condition of a company more realistically when the data is available in discrete form.

2.3 Heavy-Tail and Pareto distributions

2.3.1 Definition of Heavy-Tail Distribution

According to Raj Jain in Seleky (2022), a distribution function is said to have a thick tail if and only if for each $F(x)\mu > 0$

$$\lim_{x \to \infty} \bar{F}(x).e^{\mu x} = \infty, \tag{2}$$

on the other hand, the F(x) distribution function is said to have a *light-tailed*.

A random variable X is said to have a *heavy-tailed* if its distribution function has a *heavy-tailed*. For the record, the definition of the thick-tail distribution in equation (1) applies to the right-tail distribution, which is related to the behavior of the value of the opportunity function greater than that of $xx \to \infty$.

According to Raj Jain in Seleky (2022), the determination of exponential distribution as the boundary between thick tail and thin tail because exponential distribution serves to separate two classes of distributions that have different properties and characteristics. As an illustration, Figure 1 is given.

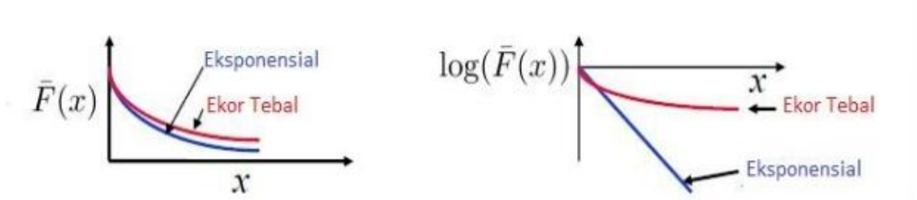

Figure 1: Comparison of Exponential Distribution and Thick Tail

2.3.2 Characteristics of the Pareto distribution

The pareto distribution comes from the name of an economist named Vilfredo Pareto (1848-1923) who observed that 80% of the wealth in Milan is owned by only 20% of the population. The Pareto distribution is also called the power-law distribution. If a dataset has a power-law distribution, then it says that the data is not sensitive to the average or standard deviation of the data or in other words, the data is not random. The Pareto Distribution opportunity density function can be expressed as follows (Saifudin, 2016):

$$f(t) = \begin{cases} \dfrac{\propto \theta^{\alpha}}{t^{\alpha+1}} \; ; \; t \geq \theta. \\ 0 \; ; \; t < \theta \end{cases} \tag{3}$$

The Cumulative distribution function for the Pareto distribution is:

$$f(t) = \begin{cases} 1 - \left(\dfrac{\theta}{t}\right)^{\propto} ; \; t \geq \theta \\ 0 \; ; \; t < \theta \end{cases}. \tag{4}$$

The survival function of the Pareto distribution is as follows:

$$S(t) = \left(\frac{\theta}{t}\right)^{\propto}, t \geq \theta. \tag{5}$$

So the hazard function is:

$$h(t) = \frac{f(t)}{S(t)} = \left(\frac{\alpha}{t}\right)^{\propto}. \tag{6}$$

For the expected value of the Pareto distribution, it is obtained:

$$E[X] = \begin{cases} \dfrac{\propto \theta}{\alpha - 1}, \alpha > 1. \\ \infty, \alpha \leq 1 \end{cases} \tag{7}$$

While the variance of the Pareto distribution is:

$$Var[X] = \begin{cases} \dfrac{\propto \theta^2}{(\alpha - 1)^2(\alpha - 2)}, \alpha > 2 \\ \infty, 1 < \alpha \leq 2 \end{cases}. \tag{8}$$

The *tail index* of the Pareto distribution is a parameter that indicates the "weight" level of the distribution tail. The smaller the value, the heavier the tail of the distribution.α, α

2.3.3 Relevance of Pareto to insurance claims data

Based on Albrecher & Kortschak (2009), in actuarial practice, the distribution of heavy tails (*heavy-tailed distributions*) proved to be very relevant for modeling insurance claims data. This is because claim data is generally characterized by large value claims that are rare but have a significant impact on the company's financial condition.

Among the various heavy-tail distributions, the Pareto distribution is one of the most popular. This distribution is able to represent the characteristics of large claims well, so it is often used to analyze *ruin probabilities* in insurance companies.

In addition to providing strong asymptotic results, the Pareto distribution also allows for the acquisition of explicit expressions for destruction probabilities. This is important not only in theoretical studies, but also to check the accuracy of estimates when applied to claims data with moderate surplus values. Practitioners often choose the classic Pareto distribution over its variations because of its simplicity as well as its compatibility with the nature of the actual claim data.

2.4 Bankruptcy Probability Model

2.4.1 Definition of Bankruptcy Probability

According to Adnan in Yunenda (2021), bankruptcy is a company's failure to generate profits, which can be in the form of economic failure (*economic failure*) or financial failure (*financial failure*). According to Widyawati in Yunenda (2021), dynamic business conditions can weaken a company's competitiveness and worsen financial conditions (Bankruptcy probability is understood as an early indication of financial stress before the company is completely bankrupt, which can cause a financial crisis (Shinta & Satyawan, 2021) and negatively impact the market reaction (Lukason & Camacho-Miñano, 2019). For this reason, the *Altman Z-Score* (Altman, 1968) widely used in predicting bankruptcy while supporting opinions *going concern* Auditor (Widyawati & Anggraita, 2013)

2.4.2 General formula for calculating *ruin probability* in discrete surplus models

Based on Shiraishi (2016), in actuarial risk theory of bankruptcy probability (*ruin probability*) is a measure of the risk that an insurance company runs short of capital so that the surplus becomes negative over a certain period of time. For a discrete surplus model, the insurance company's surplus at time can be written as:t

$$U(t) = u + p(t) - S(t), \tag{8}$$

with,
$U(t)$: the company's surplus at the time t
u : Initial capital (*initial surplus*)
$p(t)$: premiums received up to time t
$S(t)$: total claims up to time t

Bankruptcy probability is defined as the probability that the surplus will fall below zero at some point $\psi(u)$ (Grigutis & Šiaulys, 2020):

$$\psi(u) = P(\inf_{t \geq 0} U(t) < 0 \mid U(0) = u), \tag{9}$$

In discrete models with Pareto distributed claims, explicit analytical formulas are generally not available because Pareto distributions have $\psi(u) a$ *heavy-tail*, so large claims that are rare can have a significant influence on the probability of bankruptcy. Therefore, calculations are usually done by numerical or recursive methods.$\psi(u)$

2.4.3 Recursive Approach

Suppose the number of claims in the period then the probability of bankruptcy for the initial surplus can be calculated using a recursive equation:$X_t t, u$

$$\psi(u) = k = \sum_{k=0}^{\infty} P(X = k)\, \psi(u + p - k) \tag{10}$$

With limit conditions:

$$\psi(u) = 1 \ \ jika\, u < 0$$

where is the discrete probability distribution of the claim, which in the case of heavy-tail could follow the discrete Pareto distribution. This approach allows numerical calculations for various initial capital levels and $P(X = k)\psi(u)\ u$ specific time horizons, although simple analytical formulas are not available.

2.5 Hipotesis

The hypothesis of this study was formulated based on the assumption that the distribution of insurance companies' aggregate claims tends to follow the characteristics of *the heavy-tail*, which is explicitly modeled through the Pareto distribution.

a) H_1 : Thick Tail Confirmation
The estimated *tail index* parameter () of the Pareto distribution in PT XYZ's claim data is predicted to be . These results confirm the $\alpha\alpha \le 2$ *extreme heavy-tail* nature of claims, indicating that the variance of claims is theoretically unlimited.

b) H_2 : Capital Sensitivity to Risk
The probability of bankruptcy () will indicate a high sensitivity to changes in initial capital (). The marginal decline in predicted increases substantially due to the risk amplification impact of the Pareto distribution. $\psi(u) u \psi(u)$

c) H_3 : Ruin Risk Level
The probability of bankruptcy () calculated using the Pareto $\psi(u)$*heavy-tail* model is predicted to be greater than the result of the surplus model based on *the light-tail* distribution (conventional) at the equivalent capital level ().u

3. Materials and Methods

3.1 Ingredients

The object of this research is PT XYZ (AMAG). The selection of an insurance company as an object is based on the characteristics of an industry that inherently faces the risk of *heavy-tail* distributed claims, which requires a specific risk modeling approach to measure potential bankruptcy.

The data used is quantitative secondary data, extracted from AMAG's Consolidated Annual Financial Statements. Data was obtained from the official website of the Indonesia Stock Exchange (www.idx.co.id) and other related sources, covering the period ... Key data extracted and used as variables in discrete surplus models include:

a) Initial capital (*initial surplus*,) taken from the total equity at the beginning of the period,u
b) Net Premium Received (),P_t
c) Total Claims Paid ().S_t

This analysis uses a discrete surplus model combined with the assumption of Pareto (*heavy-tail*) distributed claims.

3.2 Methods

3.2.1 Discrete Surplus Modeling

The first step in this modeling is to determine the average net premium value (), which is the expected value of the difference between premiums and claims per period. This value represents the *ptrend* of the net surplus received by the company on average. The change in the company's capital surplus is modeled discretely from one period to the next, where the calculation takes into account the previous period's surplus, incoming premiums, and total claims paid.

3.2.2 Estimated Claim Parameters

The parameters of the Pareto distribution, namely scale () and tail () are estimated from the total claims paid () data using the $\theta\alpha S_t$*Maximum Likelihood Estimation* (MLE) Method. The MLE method is applied to obtain an efficient estimate of *the value of the tail index* (). This value is an important indicator that will be used to verify the research hypothesis related to the characteristics of $\hat{\alpha}$*the heavy-tail* in the claims faced by the AMAG.

3.2.3 Calculation of Bankruptcy Probability

The probability of bankruptcy () is calculated numerically using a recursive approach. This approach was chosen because the discrete surplus model with heavy-tail Pareto distributed claims $\psi(u)$ does not have an explicit analytical solution. Before the calculation, the continuous Pareto distribution is dissected to the discrete mass probability, then the value is used in an iterative calculation for . The entire numerical calculation and scenario analysis is performed with the help of Python syntax $P(X = k)\psi(u)$ to measure the risk sensitivity of bankruptcy (*ruin*) to initial capital variations ().u

4. Results and Discussion

4.1 Results of Pareto Distribution Parameter Estimation

The Pareto distribution is used to describe the distribution of the value of insurance claims that are heavy-tail. Based on the results of parameter estimation using the Maximum Likelihood Estimation (MLE) method, the following parameter values were obtained:

Table 1: Results of Estimation of Pareto Distribution Parameters

Parameter	Symbols	Estimated Value	Remarks
Scale	x_m	1,000,000	Minimum claim value
Tail Index	α	2.47	Measuring the severity of the distribution tail
Expetasi	$E[X]$	678,000	Average claim value
Variansi	$Var[X]$	6.2×10^{11}	Claim data dissemination rate

A *tail index* value of 2.47 indicates that claims data is *heavy-tail*, where most claims are of small value but there is a chance of large value claims. The high value of variance also reinforces that this distribution has a wide spread, making it suitable for modeling claims with extreme risk. The Pareto distribution with these parameters is then used in the simulation of the probability of bankruptcy in the next subchapter.

4.2 Bankruptcy Probability Simulation

The simulation was carried out to assess the influence of initial capital, premiums, and the frequency of claims on the *ruin probability* of bankruptcy in insurance companies. Probability values are calculated based on a discrete time surplus model using pre-estimated Pareto parameters.

Table 2: Bankruptcy Probability Simulation Results

Initial Capital	Prize	Frequency of Claims	Probability of Bankruptcy
5,000	2,000	5	0.381
10,000	2,000	5	0.178
15,000	2,000	5	0.087
20,000	2,000	5	0.046
10,000	3,000	5	0.061
10,000	2,000	8	0.264
15,000	3,000	8	0.093

The simulation results show that an increase in initial capital and premiums can significantly lower the probability of bankruptcy. On the other hand, the increase in the frequency of claims has a major effect on the increase in the risk of bankruptcy. For example, at an initial capital of 10,000, an increase in the premium from 2,000 to 3,000 lowers the probability of bankruptcy from 0.178 to 0.061. However, when the frequency of claims increased from 5 to 8, the chance of bankruptcy rose from 0.178 to 0.264. These findings demonstrate the importance of a balance between premium income and claims risk exposure in maintaining the company's financial stability.

4.3 Effect of Distribution on Risk Estimation

To test the sensitivity of the model, a comparison was made between the Pareto distribution (*heavy-tail*) and the exponential distribution (*light-tail*) with the same basic parameters.

Table 3: Bankruptcy Probability Simulation Results

Initial Capital	Pareto	Exponential
5,000	0.381	0.239
10,000	0.178	0.092
15,000	0.087	0.034
20,000	0.046	0.011

The value of the probability of bankruptcy with the Pareto distribution is consistently higher than the exponential distribution at all levels of initial capital. This suggests that heavy-tail distribution-based models are more sensitive to large and rare claims, while such exponential light-tail models tend to underestimate the potential for extreme risk. Thus, the use of Pareto distribution is considered more representative in modeling the characteristics of claims in the insurance industry.

4.4 General Discussion

Overall, the results of the analysis show that:

a) Initial capital and premiums are the main factors that lower the risk of a company's bankruptcy.
b) The frequency of claims and the heavy-tail nature of Pareto contribute greatly to the increased probability of bankruptcy.
c) The Pareto distribution provides a more conservative and realistic result than the exponential distribution, as it is able to illustrate the impact of extreme claims on financial stability.

These findings support the results of previous research that states that a Pareto-like heavy-tail distribution risk model is more suitable for use in insurance contexts, where extreme claim values, although rare, have a significant influence on the probability of bankruptcy**.**

5. Conclusion

The results show that the discrete time surplus model with Pareto distribution can accurately represent the risk of bankruptcy of an insurance company. Initial capital and larger premiums have proven to be effective in reducing the risk of bankruptcy, while the increased frequency of claims and the heavy-tail nature of the Pareto distribution increase the risk significantly. This model provides a realistic and conservative approach to measuring the financial resilience of insurance companies to potential extreme claims. This study provides a methodological contribution to the actuarial literature by demonstrating that the application of a discrete-time surplus model based on a heavy-tail

Pareto distribution is able to represent the bankruptcy risk of insurance companies more conservatively and realistically than a model based on a light-tail distribution. The analysis results confirm that the characteristics of extreme claims have a dominant role in determining the probability of bankruptcy, so that the heavy-tail approach is important as a basis for risk management decisions and the formulation of solvency policies in the insurance industry.

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