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Application of Economic Mathematics in Periodical Installation System

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Abstract

This paper provides financial planning aspect by modeling the impact of mortgage rate changes on the size of payments for ARMS. This study uses a simulation approach to model the choice between a fixed rate mortgage (FRM} and an adjustable rate mortgage (ARM). Our simulations help assess the risks and benefits of choosing an ARM rather than a FRM. We represent the risk of the ARM with distributions of present value cost for a variety of mortgage life periods.

Keyword: Annuity, fixed rate mortgage (FRM}, adjustable rate mortgage (ARM)

1. Introduction

The periodic installment system is a series of payments made at certain intervals. The periodic installment system is a common problem in the economy and banking (Bidabad, 2018; Rathore, et el., 2016; Grassa, 2013). Mortgage payments, car loan payments, house rentals are examples of some of the periodic installments. There are so many problems in the economy, especially in financial matters related to the periodic installment system (Calhoun & Deng, 2002).

Periodic installment systems are used by credit agencies such as real estate and insurance installments. The periodic installment system can benefit two parties, namely the party and the credit recipient (Ramadhani & Al Imron, 2020; Myranika, 2021). For lenders, lenders will benefit from loan interest. As for credit recipients, they will get relief to repay the credit.

Periodic installments also have a lot of problems, such as the amount of money that must be paid each year or the length of time it takes to make payments (Dayanand & Padman, 2001). To solve this problem, we need to find the right method. Therefore, the author is interested in discussing the periodic installment system by applying economic mathematics to solving problems with the periodic installment system for home mortgages in the economy Abad-Segura & González-Zamar, 2020; Martins & Olgin, 2019).

2. Regular Installment System

In the periodic installment system, several terms and methods are known, such as Truth in Lending or what is often known as Regulation Z, amortization, mortgage, Fixed rate mortgage (FRM}, Adjustable rate mortgage (ARM) and companion elements that are charged in the system. as an introduction in the discussion the author uses the annuity model (Green, 2013; Lea, 2010).

2.1. Truth In Lending

The main objective of the Truth in Lending regulation is to regulate lenders to make a fair and proper approach in providing loans to consumers (62 FR 5183, Feb. 4, 1997) (Peterson, 2003). The regulation provides two key values of the financial approach called the fee charge and the annual rate (known as the APR). Some of these regulations are:

Installment payments are given by.

$$R = \frac{L+K}{n} \tag{1}$$

Because the value of this installment payment must be equal to the principal amount of the loan, we have an equation with value.

$$R_{a_{\overline{n}|j}} = L \tag{2}$$

For value with attention to. Then the annual rate is

$$i = mj \tag{3}$$

2.2. Mortgage

Real Estate Mortgage is one example where the payment resembles an installment loan (62 FR 5183, Feb. 4, 1997). Then we have the following relationship.

$$i` = 12\,j`. \tag{4}$$

The annual interest rate on the loan is

$$R = \frac{L}{a_{\overline{n}|j`}}. \tag{5}$$

The total fee for Truth in Lending purposes is the loan amount minus the cost on settlement to be reflected, ie.

$$L^* = L - Q. \tag{6}$$

The fee is equal to the difference between the total payments to be made and the amount financed for the purpose of the lending Truth in Lending, ie.

$$K = nR - L^*. \tag{7}$$

In order to obtain the interest rate per month for Truth in Lending we get the following equation with

$$Ra_{\overline{n}|j} = L^* \tag{8}$$

Finally, Truth in Lending's annual percentage rate is

$$i = 12\,j\,. \tag{9}$$

The loan amortization schedule involves installments over a relatively short period of time. At the beginning of the payment almost all of it is interest, while the final payment is all of the main loan.

Where

L : principal loan balance after initial payment
K : cost burden
R : installment payments
m : number of payments per year
n : total payment amount of the loan
i : total annual rate
j : interest rate per payment period
Q : payment made on settlement that must be reflected in the APR
L^* : total cost in the Truth in Lending objective, which is reflected by
j' : monthly interest rate on the loan
i' : annual interest rate on the loan

3. Types of mortgage loans

3.1. Fixed Rate Mortgage

Fixed rate mortgage is a mortgage in which the installment interest does not change during the installment period, and is set at the beginning of the installment agreement (Ampofo, 2020). In fixed rate mortgages there are penalties or fines if you end installments that are not at the specified time. The advantage of a fixed rate mortgage is that it will be easier to find out the size of the mortgage payment over a predetermined period of time. The disadvantage of this type is that at the beginning the penalty payment is given, and it will continue to increase in value at the end of the installment period, the value of which is close to 6 months of installment payments.

3.2.1. Fixed Rate Mortgage Mechanism

At the end of the installment, the present value (v) in the annuity must be equal to the amount of funds provided by the seller to the consumer.

$$A = v_0 \tag{10}$$

At time 0 it is known that the value of the mortgage (R) is equal to the value of the amount of funds to be paid. Assume the mortgage amount equals the amount owed.

So, the mortgage value equation

$$L = R * \frac{\left(1 - \frac{1}{(1 + j/12)^n}\right)}{j/12} \tag{11}$$

So if the contract value is 12%, with a term of 30 years (360 months) and the monthly installments are $850, the mortgage amount will be $82,635.58

$$82{,}635.58 = 850 * \frac{\left(1 - \frac{1}{\left(1 + .12/12\right)^{360}}\right)}{.12/12}$$

3.2.2. Calculation of monthly installment payments.

$$L = R * \frac{\left(1 - \frac{1}{(1 + j/12)^n}\right)}{j/12} \tag{12}$$

So for a mortgage value of $ 200,000 with a term of 30 years, and 12% interest, the monthly installments will be $ 2,057.23.

$$2057.23 = 200{,}000 * \frac{(.12/12)}{\left(1 - \frac{1}{\left(1 + .12/12\right)^{360}}\right)}$$

This equation works at every point every time.

3.2.3. Amortization calculation mechanism

General (if payments are made on time), the seller doesn't charge interest, only the principal installment amount. The interest value is obtained by multiplying the first installment amount by the monthly interest.

$$\text{Principal interest} = \text{opening balance} * j/12 \tag{13}$$

Mortgage installments can be obtained by reducing the principal interest, namely:

$$\text{Mortgage repayments} = \text{R} - \text{principal interest} \tag{14}$$

So we get a table for a mortgage for 30 years, at 12% interest, on a $200,000 mortgage as follows.

Table 1: Table of the relationship between the main installment, interest and installment value calculated every month for 30 years of mortgage

Installment amount	$200,000,00	many payments	360	interest rate	12.00%
month	beginning balance	monthly payments	tree flower	instalment hipotek	ending balance
1	$200,000.00	2057.225194	$200.00	$57.23	$199,942.77
2	$199,942.77	2057.225194	$199.43	$57.80	$199,88.98
3	$199,88.98	2057.225194	$198.85	$58.38	$199,826,60
:	:	:	:	:	:
358	$4.773.69	1623.161434	$47.74	$1.575.42	$3.198.27
359	$3.198.27	1623.161434	$31.98	$1.591.18	$1.507.09
360	$1.507.09	1623.161434	$16.07	$1.607.09	$0.00

Mortgage price calculation mechanism

$$\text{Mark hipotek} = \text{R} * \frac{\left(1 - \frac{1}{\left(1 + r/12\right)^n}\right)}{r/12}$$

This equation is the basic equation used to calculate the balance, only the difference is used as the market value, to replace the interest rate.

3.2. Adjustable Rate Mortgage

In adjustable rate mortgages, the interest rate is set every month of payment, after being adjusted for the rise or fall of the market index or interest rates in effect during the specified change period (Boyd, et al., 2005). From the borrower's perspective, adjustable rate mortgages are more attractive, because the initial interest rate on monthly payments is lower than the interest rate charged on a fixed rate mortgage type by adjusting to the prevailing interest rate.

Table 2: Time period of Adjustable rate mortgage

	Mortgage payment intervals	Time	Amortization
ARM 3 month	3 month	30 year	30 year
ARM 6 month	6 month	30 year	30 year
ARM 1 year	1 year	30 year	30 year
ARM 2 year	3 year	30 year	30 year

The interest rate on an adjustable rate mortgage is calculated as the index rate plus a fixed margin, which is the margin indicated on the original mortgage agreement.

3.2.1. Mechanism of Adjustable Rate Mortgage

In calculating the ARM there are "four rules" of loan payments and their balance (balance)

- The first rule: interest is deferred in each payment according to the interest rate that has been equated with the value of the unpaid debt (which is known as the outstanding loan balance or OLB) at the end of the payment period.

$$INT_t = (OLB_{t-1})r_t$$

- The second rule: the main loan is amortized (repaid) on each payment equal to the total payment (net of expenses and penalties) minus any outstanding interest

$$AMORT_t = PMT_t - INT_t$$

- Rule three: the main outstanding balance after each payment is equal to the previous balance minus the outstanding debt.

$$OLB_t = OLB_{t-1} - AMORT_t$$

- Fourth rule: the initial principal debt is the same as the initial debt contract in the loan agreement

$$OLB_t = L$$

Where :

L : initial debt contract in the loan agreement;

r_t : interest rate according to the agreement for payments during period t

INT_t : interest during period t;

$AMORT_t$: main debt paid during period t ;

OLB_t : main debt balance that has not been paid during period t ;

PMT_t : loan value in period t;

Loan interest:

$PMT_t = INT_t$ or equivalent to $OLB_t = L$ for each t.

Example: interest payments on a loan and an interest component of $1,000,000, 12%, 30-years, payments are made monthly.

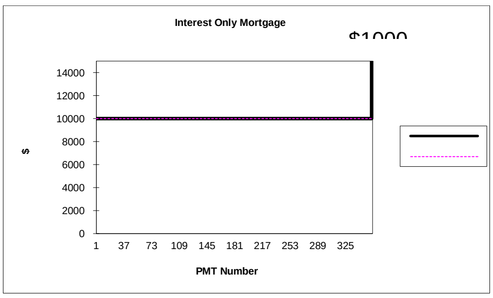

Figure 1. The relationship between interest and the principal loan value

3.2.2. Adjustable rate mortgages (ARM):

$r_t \neq r_{t+s}$ for every s and t

How to calculate ARM payments and balance:

Step 1: Formulate the interest rate at each interval: Index, Margin and caps used.

Where r is fixed in each interval as:

Min from : $index + margin$

: r before + interval cap,

: initial r + full stamp

Step 2: Formulate the monthly payments at each interval:

The PMT in each interval is determined as the OLB (at the end of the previous interval) divided by the PVIFA at the new interest rate, provided that it is equal to the number of periods stated on the original mortgage regarding the interval. (VBM caps limit can be changed to PMT

Step 3: use the "4 rules" of mortgage payment and balance sheet calculations.

$INT_t = r_t(OLB_{t-1})$

$AMORT_t = PMT_t - INT_t$

$OLB_t = OLB_{t-1} - AMORT_t$

$OLB_0 = L$

Example: Adjustable rate mortgage (ARM) payments and interest components: $1,000,000, 9% prime interest, 30-years, payments made monthly; additional 1 year interval in Figure 2.

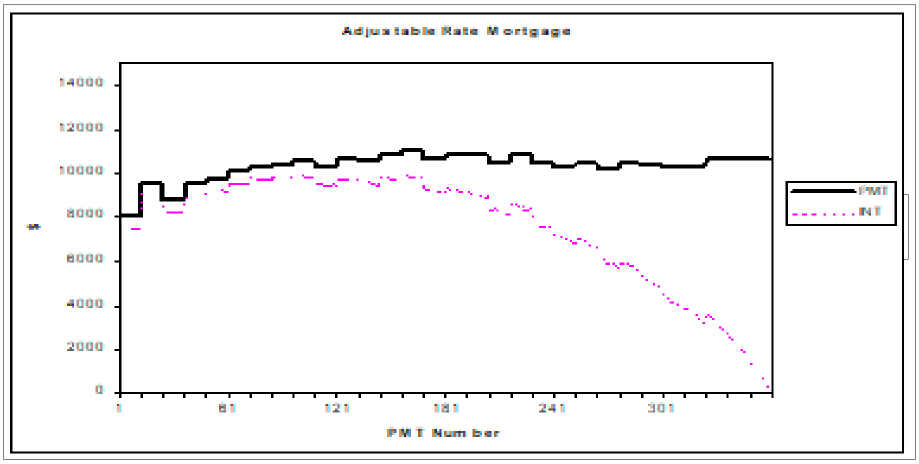

Figure 2. Graph of payments made monthly

Month	Rule 3 & 4 OLB(start):	PMT:	Rule 1 INT:	Rule 2 AMORT:	Rule 3&4 OLB(end):	Level used
0					1000000	
1	1000000	8046.23	7500	546.23	999454	0.09
2	999454	8046.23	7495.9	550.32	998903	0.09
3	998903	8046.23	7491.78	554.45	998349	0.09
...	...	...	...	...	...	...
12	993761	8046.23	7453.21	593.02	993168	0.09
13	993168	9497.14	9099.82	397.32	992771	0.1099
14	992771	9497.14	9096.18	400.96	992370	0.1099
...	...	...	...	...	...	...
24	988592	9497.14	9057.89	439.25	988152	0.1099
25	988152	8785.78	8247.66	538.12	987614	0.1002
26	987614	8785.78	8243.16	542.62	987072	0.1002
...	...	...	...	...	...	...
358	31100	10605.44	356.65	10248.79	20852	0.1376
359	20852	10605.44	239.12	10366.32	10485	0.1376
360	10485	10605.44	120.24	10485.2	0	0.1376

3.2.3. Elements of an Adjustable Rate Mortgage

a) Additional Intervals
b) Indices
c) Margins
d) Caps & Floors (in pmt, contract value rate)
e) Value of the main contract rate
f) Prepayment Privileges
g) Conversion

In general, adjustable rate mortgages have an upper limit on the amount of increased payments and/or interest rates, which are called caps. Most of the adjustable rate mortgage caps increase annually or at the time limit of the loan in Figure 3.

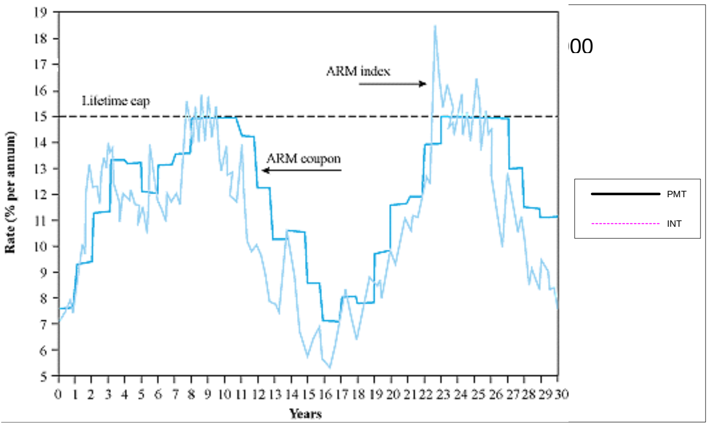

Figure 3 The effect of caps rates on mortgage values

4. Conclusion

In choosing the method of periodic installments between Fixed rate mortgages (FRM} and Adjustable rate mortgages (ARM) one must pay attention to the interest charged, the length of the installments and especially the ability to pay periodic installments considering that the longer the repayment period, the higher the interest charged.

The material for predicting the number of visitors to the Ujung Genteng Beach area of Sukabumi uses the Holt-Winter method in the form of field data from a recap of the number of tourist visitors at the entrance to the Ujung Genteng beach attraction every month from January 2017 - February 2020. Data analysis was carried out with the help of Ms. software. Excel and R software.

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