



The Use of Quasi Monte Carlo Method with Halton Random Number Sequence in Determining the Price of European Type Options (Case in PT Telekomunikasi Indonesia Stock's)

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Abstract

An investor must be wise in managing the funds he has to carry out investment activities. Investors can use options as an alternative to investing because they can increase profits and avoid investment risks. Options are one of the most widely used derivative products. The main problem when entering into an option contract is determining the right price to be paid by the option buyer to the option seller. This research was made to determine the price of European-type stock options. Case studies on PT Telekomunikasi Indonesia, Tbk shares in the 2021-2022 period. The analysis was performed using the Quasi-Monte Carlo method with Halton's random number sequence. Based on the results of this study, it is expected to be a consideration in deciding to buy European-type stock options at PT Telekomunikasi Indonesia, Tbk.

Keywords: Investment; european option; Quasi-Monte Carlo; random number sequence

1. Introduction

Investment is an activity that carries unexpected risks, so an investor must be wiser in managing the funds to be invested. Derivative products are one alternative investment that an investor can use to control risk. The function of derivative products is to increase profits and avoid investment risks. Option contracts are an example of derivative products (Higham, 2004). An option contract is an agreement or contract entered into by the seller of the option with the buyer in which there is a right, not an obligation, to sell or buy certain shares at a price and time that has been agreed upon and guaranteed by the seller of the option from the buyer.

Investors can use options to prevent the impact of declining market prices on their portfolios and to speculate on stock price movements. The issue of concern when entering into an option contract, the issue of concern is how to determine the exact price that the option buyer must pay to the seller (Zubaedi et al., 2022). Methods and models that can be used to determine option prices are the Black Scholes model, Monte Carlo simulation, Quasi-Monte Carlo Method, Binomial Option Pricing Model, and so on (Sumampow et al., 2020).

In previous research, Ratnasari et al. (2017) examined the calculation of binary option contract values for cocoa commodities using Faure random sequences. The results of his research were the Quasi-Monte Carlo method with Faure random numbers more convergent and closer to zero than the Standard Monte Carlo method. Syam et al. (2019) predicted prices for European options using the Monte Carlo method. In his research, he stated that the predicted values for European options would converge and be better if they carried out many iterations during the simulation using the Monte Carlo Standard method.

This study aims to determine the European-type option price on PT Telekomunikasi Indonesia Tbk. The price of the option is determined using a Quasi-Monte Carlo simulation method with a random Halton number sequence. The

contribution of the results of this study is expected to be a material consideration for investors' decision-making in purchasing stock options, especially for PT Telekomunikasi Indonesia Tbk stock options.

2. Literature Review

2.1 Option

Stock options are derivative products that can be an alternative for investors to make investments because they can reduce the impact or risk of stocks occurring. Options can be grouped into two: Call option or call option is an agreement or arrangement whereby the option buyer has the right to buy a number of shares at a certain price and period of time from the option seller (call). According to Bezberodov (2016), the notation of a call option is C (call option). The difference between the stock price and the predetermined strike price is the profit earned from the call option, expressed as equation (2.1).

$$C = \text{Max} (S_t - K, 0), \quad (2.1)$$

with:

C : call option,

S_t : stock price at the time t ,

K : The agreed price according to the agreement (strike price).

A put option or put option is an agreement or agreement whereby the option buyer has the right to sell several shares at a certain price and period of time from the option seller. The notation of a put option is P (put option). The difference between the strike price and the current stock price is the profit earned from the put option, expressed as equation (2.2).

$$P = \text{Max} (K - S_t, 0). \quad (2.2)$$

Based on the exercise right and time period, the options are divided into two, namely:

1. European Option

This type is an option that is exercised only on the expiration date.

2. America Option

This type is an option that is exercised before or at the expiration date

2.2 Pay-off

The pay-off is the benefit the option owner gets when exercising the option. For European call options, if at maturity, the price of S_t shares are greater than the strike price (K), the pay-off will have a value greater than 0. Meanwhile, for European put options at expiration, the price of S_t shares are lower than the strike price (K), and the pay-off will have a value greater than 0 (Hull, 2009). A put option has a pay-off function written in equation (2.1), while a call option is written in equation (2.2).

2.3 Estimated Return and Stock Price Volatility

A share certificate is a form of company ownership in the form of a document. If the company experiences a profit, the investor is entitled to a share of the profits in accordance with the proportion of their ownership. The results obtained from these investments are called stock returns (Zubir, 2013). In carrying out investment activities, investors will choose a form of investment that provides high returns, which is called the rate of return. The value of stock returns can be calculated by equation (2.3).

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right), \quad (2.3)$$

dengan:

S_t : stock price at the time t ,

S_{t-1} : stock price at the time $t - 1$.

Variance is a measure of the spread of data used to determine how far the spread of values from observations results in the average (Putri, 2018). The smaller the value of a variance, the stock return is closer to the expected return value. Conversely, the greater the variance value, the further away from the expected return value. Equation (2.4) is used to calculate the expected return value.

$$E[r] = \frac{\sum_{i=0}^n r_i}{n}. \quad (2.4)$$

As for calculating the variance, use equation (2.5).

$$Var[r] = \frac{\sum_{i=0}^n E[(r_i - E[r])^2]}{n-1} \quad (2.5)$$

with:

r_i : stock returns,

$E[r]$: average stock return,

n : stock data.

Volatility is a measure of the uncertainty that affects the option price of a stock exchange. To calculate the volatility value, equation (2.6) is used.

$$\sigma = \frac{1}{\sqrt{\tau}} \sqrt{Var[r]}, \quad (2.6)$$

where T is the maturity time.

2.4 Stock Price Movement Model

The basic model used to calculate stock price movements resembles the motion of gas particles that move randomly or Brown Motion. Suppose $\{S(t), t \geq 0\}$ is the stock price at one time, with $t \geq 0$.

According to Capinski (2003), a stochastic process S_t with $t \geq 0$ is said to be a Brownian motion process if:

1. S_t is a continuous function and $S(0) = 0$.
2. $\{S_t, t \geq 0\}$ has free increment and stationary increment.
3. $S_t > 0, S_t$ normally distributed with a mean of 0 and a variance t .

Brownian Motion has an average value of zero, while stock prices in a certain period usually move with a certain growth rate which is described by the drift factor (μ). Brownian Motion with drift can be defined as equation (2.7)

$$S_t = \mu_t + W_t, \quad (2.7)$$

with W_t is the Brownian Motion Standard process for describing price growth rates.

The characteristics of the movement of a process differ from one process to another, some processes are relatively calm, and some are fluctuating. In Brownian Motion, a constant noise scale (σ) can be added to represent the movement so that the model of Brownian Motion can be shown as equation (2.8)

$$S_t = \mu_t + \sigma W_t \quad (2.8)$$

In Brownian Motion with the Drift and Noise scales, there is a weakness as a model for changes in stock prices because it is still possible to have a negative value. Therefore, there is a variation of Brownian Motion, which is developed into Geometric Brownian Motion. According to (Tse, 2009), the stock model can be defined by equation (2.9)

$$S_t = S_0 \exp(\mu_t + \sigma W_t) \quad (2.9)$$

2.5 Quasi-Monte Carlo

According to Mukhti et al. (2018), the Monte Carlo method is an algorithm for solving a problem with a random process (randomization) in the form of probability simulation. The probability used to estimate is by generating random numbers. Monte Carlo simulation is carried out in several repetitions of the simulation. The number of repetitions of the simulation is done by generating as many random numbers as the simulation. The Monte Carlo method using standard normally distributed random numbers can be shown as equation (2.10)

$$S_t = S_0 \exp \left[\left(\mu - \frac{\sigma^2}{2} \right) T + \sigma \epsilon \sqrt{T} \right], \quad (2.10)$$

with $S(t)$ is the stock price at the time t and ϵ is a normally distributed random number ($\epsilon \sim N(0,1)$)

The Quasi-Monte Carlo method is a Monte Carlo method that uses a sequence of random numbers as a substitute for random numbers. Random number sequences can be divided into Van der Corput, Faure, Halton, and Sobol random number sequences. This sequence of random numbers is used to generate a representative sample of the probability distributions simulated in practical problems. These random number sequences often referred to as low discrepancy sequences, can improve the performance of Monte Carlo simulations, shorten computation time, and provide higher accuracy. The Quasi-Monte Carlo method is generally used as an alternative to the Monte Carlo method in multidimensional numerical integration problems.

An examination of the various errors of estimation in providing convincing evidence that what should be applied to Quasi-Monte Carlo integration is a collection of nodes with very small differences. Limited or infinite sequences with this condition are called low-discrepancy sequences, and this term refers to the quasi-random points or quasi-random numbers in the one-dimensional state.

3. Materials and Methods

In this research, the object is the stock option price at PT Telekomunikasi Indonesia Tbk. The share price used is the daily closing price (close price) in 2021-2022. To obtain closing price data, this study uses secondary data on stock prices listed on the IDX and obtained through the website www.yahoofinance.com. Calculating option prices using the Quasi-Monte Carlo Method with Halton random numbers with the help of Microsoft Excel and R studio software. This study uses a quantitative and applicative approach through case studies, namely the price of European-type stock options at PT Telekomunikasi Indonesia Tbk. There are several steps in conducting this research, namely:

1. Collect historical stock price data through the website www.yahoofinance.com.
2. Calculating return using equation (2.3), calculating expected return using equation (2.4), calculating variance using equation (2.5), and determining stock volatility using equation (2.6).
3. Determine the value of the initial stock price (S_0), stock volatility (σ), risk-free interest rate (i), maturity date (T), and exercise price (K) obtained from historical data. Uses predefined K and T values
4. Simulating PT Telekomunikasi Indonesia's stock options prices using the Quasi-Monte Carlo Method using Halton's random number sequence.

There are steps to simulate Quasi-Monte Carlo, namely:

- a. Simulating the stock price S by using equation (2.10).
- b. Calculating the option profit (pay-off) for Call Options using equation (2.1) and Put Options using equation (2.2).
- c. Generate Halton random number sequence
- d. Repeat the first and second steps N times which is a Quasi-Monte Carlo simulation process.

4. Result and Discussion

4.1 Stock price

In this study, 252 data on daily closing stock prices (close prices) of PT Telekomunikasi Indonesia, Tbk was used. The data used was obtained from www.yahoofinance.com. The graph of daily closing stock price data is shown in Figure 1.



Figure 1: Grafik harga saham penutupan harian

Based on Figure 1, the daily closing share price data at PT Telekomunikasi Indonesia, Tbk in the 2021-2022 period experienced fluctuations, but not too significantly, and there was a trend or tendency to increase even though in several periods

4.2 Normality test

In this section, a normality test of PT Telekomunikasi Indonesia's daily closing price data is carried out to determine whether the data is normally distributed by making a Q-Q Plot using the R Studio software. The Q-Q Plot of PT Telekomunikasi Indonesia, Tbk shares is shown in Figure 2.

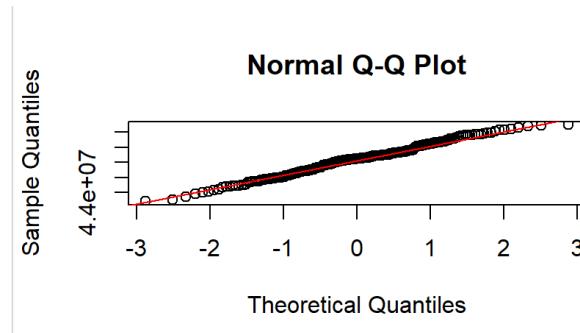


Figure 2: Normal Q-Q Plot PT Telekomunikasi Indonesia, Tbk

Based on Figure 2, PT Telekomunikasi Indonesia's daily closing share price data is normally distributed. This can be seen from the distribution of PT Telekomunikasi Indonesia's daily closing share price points which are in a linear line.

4.3 Return, Expectations, and Volatility

The Log Return value of shares of PT Telekomunikasi Indonesia, Tbk is calculated from the daily closing price of the shares. For $t = 1$; $S_0 = 3590000000$ and $S_1 = 3560000000$, so obtained:

$$r_1 = \ln\left(\frac{3560000000}{3590000000}\right) = -0.008392$$

The calculation is repeated for daily closing stocks during the period September 22 2021 to September 30, 2022. After calculating stock price returns, it is then used to calculate expected returns, so we get:

$$E[r] = \frac{0.216997}{252} = 0.000865$$

So the expected value of PT Telekomunikasi Indonesia, Tbk's stock return is 0.000865.

After the expected return value is obtained, it can be used to calculate the variance value using equation (2.5) as follows:

$$Var[r] = \frac{\sum_{i=0}^{252} (0.000086 + 0.000001 + 0.000148 + \dots + 0.000058 + 0.000013)}{252 - 1}$$

$$Var [r] = \frac{0.067560}{251} = 0.000269$$

So the stock variance value of PT Telekomunikasi Indonesia, Tbk is 0.000269. Based on the variance value of stock returns, it is then used to calculate volatility using equation (2.6) assuming trading days for one year, namely $\tau = \frac{1}{252}$, so that the volatility value can be calculated as follows:

$$\sigma = \frac{1}{\sqrt{\frac{1}{252}}} \sqrt{0.000269}$$

$$\sigma = \frac{1}{0.062994079} (0.016406159)$$

$$\sigma = 0.260439701$$

So, the value of stock price volatility for 252 days in one year is 0.260439701.

4.4 Option value calculation

In calculating the option price in this study, the reference interest rate issued by Bank Indonesia is used as a risk-free interest rate, namely $i = 5\%$. This data was obtained from the official website of Bank Indonesia (www.bi.go.id). Then based on information on the share price of PT Telekomunikasi Indonesia Tbk, which was traded from 22 September 2021 to 30 September 2022, or the equivalent of 252 trading days, the strike price (K) is 4400.

Initial stock price (S_0) is IDR.4460, volatility (σ) is 0.260439701, maturity time (T) is 1, and return expected value (μ) is 0.000865. Then determine the number of generations of simulation data as much as $n = 100$ to $n = 1000000$. The simulation results obtained for call options are given in Table 1.

Table 1. Call option values

N	Quasi Monte Carlo call option	
	value	standard error
100	550.9974	78.73976
200	568.1497	57.59193
300	575.2535	47.97741
400	579.1878	41.74460
500	584.0806	37.78416
600	584.0126	34.56227
700	585.3098	31.99137
800	589.2888	30.26049
900	589.3203	28.56225
1000	589.7251	27.09894
2000	593.2744	19.34059
3000	594.1895	15.84779
4000	594.8302	13.74924
5000	595.3228	11.24547
6000	595.6878	10.42166
7000	595.7998	9.75711
8000	596.2725	9.20497
9000	596.3318	8.74086
10000	596.2661	8.33111
1000000	597.5930	0.6208375

From Table 1 above, it can be seen that the changes obtained are where the greater the number of simulations performed, the smaller the standard error. This shows that the value obtained has led to a value so that the more simulations are carried out, the smaller the deviation will be. In the 1000000th simulation, the standard error obtained is close to zero, so it can be concluded that the call option's value is 597.593 with a standard error of 0.6208375. Meanwhile, based on Table 1. a graph is created to see the simulation results of the value of the call option shown in Figure 3.

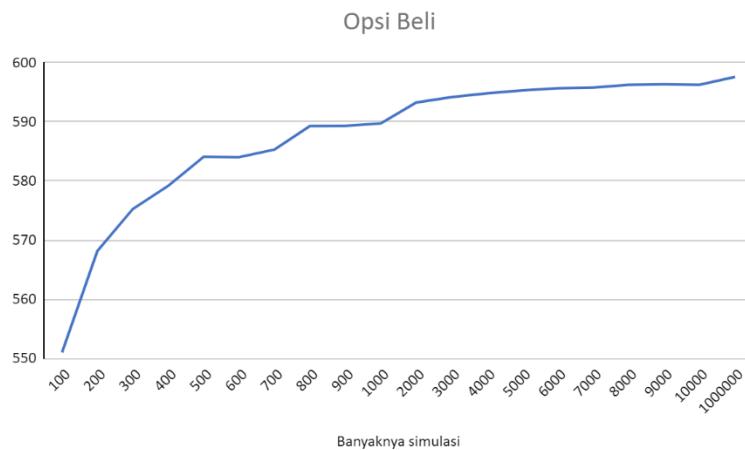


Figure 3. Graph of Buy Options

Based on Figure 3, it can be seen that the more simulations are carried out, the resulting option values converge to a value. While the simulation results obtained for put options are given in Table 2.

Table 2. Put option values

Quasi Monte Carlo		
n	put option value	standard error
100	334.8044	49.02342
200	329.8203	34.53039
300	329.0159	28.18764
400	326.6050	24.32026
500	325.2063	21.67290
600	326.3449	19.86037
700	324.2230	18.28043
800	324.9012	17.17877
900	325.4008	16.20663
1000	324.9383	15.35129
2000	323.8595	10.83036
3000	323.9088	8.84990
4000	323.6736	7.65949
5000	323.5589	5.78924
6000	323.4400	5.41296
7000	323.4289	5.10403
8000	323.2639	4.84009
9000	323.4139	4.61755
10000	323.3934	4.42049
1000000	323.0299	0.3710202

Table 2 above shows that the changes obtained are where the greater the number of simulations performed, the smaller the standard error. This shows that the value obtained has led to a value so that the more simulations are

carried out, the smaller the deviation will be. In the 1000000th simulation, the standard error obtained is close to zero, so it can be concluded that the value of the put option is 323.0299 with a standard error of 0.3710202. Meanwhile, based on Table 2. a graph is made to see the simulation results of the value of the put option shown in Figure 4.

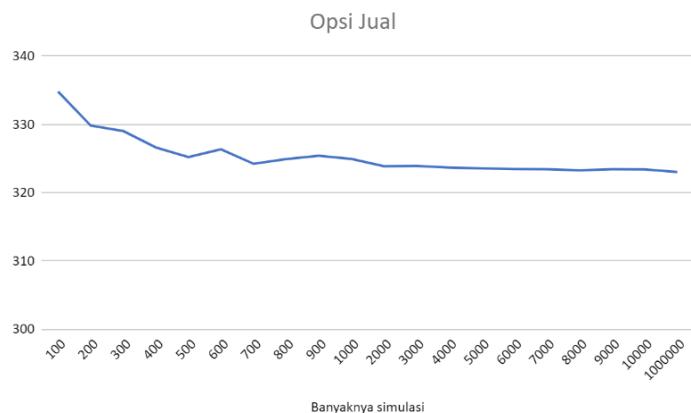


Figure 4. Graph of Put Options

Based on Figure 4 it can be seen that the more simulations are carried out, the resulting option values converge to a value.

5. Conclusion

The research results on PT Telekomunikasi Indonesia's daily closing share price data on September 22, 2021-September 30, 2022, or the equivalent of 252 trading days, show that the share price data is normally distributed. Based on the results of the calculation of the initial stock price (S_0) of IDR.4460, volatility (σ) is 0.260439701, the expected value of the return (μ) is 0.000865, maturity (T) is 1, interest rate (i) is 5%, and the strike price (K) is 4400. With $n = 100$ to $n = 1000000$ simulations. The call option price uses the Quasi-Monte Carlo method with Halton's random number sequence of IDR. 597.593 with a standard error of 0.6208375 and a put option price of IDR.323.0299 with a standard error of 0.3710202. The more simulations that are carried out, the smaller the deviation, and the results of the option calculations will converge to a certain value.

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